# An attempt at a typology of karst lakes in the Połaniec Basin (Małopolska Upland, Poland)

**Abstract**
The main goal of the paper is to attempt a typology of karst lakes in the Połaniec Basin (Małopolska Upland). The typology was conducted on the basis of a dependence analysis of several essential morphometric parameters of lake basins. The considered data comprised 23 lakes with respect to15 morphometric features. The correlation analysis, mainly in a group of lakes that are located in single karst sinkholes, revealed that length and width of basins are distinctly associated features. It is also noticeable that basin shape determines lake volume, even though pools, that are characterised by similar water content volume, may differ in area. Moreover, an increase in maximum depth of basins does not necessarily imply any grow in their volume. Likewise, there is no prevalent dependence between basin area and maximum depth.

The cluster analysis, among reasonable indications, singled out generally divisions of the considered lakes into two sets. One of the sets comprises Duży Staw and Dziki Staw, while the other consists of all the remaining lakes. Less frequent divisions into three indicated Duży Staw, Dziki Staw, and Czwarty Staw as leading lakes. Divisions into 19–22 clusters were also suggested, however it does not seem to be reliable. As a consequence, the cluster analysis elucidate that Duży Staw and Dziki Staw stand out the most from the other lakes. This remainder constitutes rather close to each other, but not an ideally uniform group of lakes.

**Keywords**: karst lakes, morphometric parameters of lakes, correlation analysis, cluster analysis, Połaniec Basin

## 1. Introduction

The occurrence of lakes above 1 ha is not frequent in the Polish Uplands (Choiński 1995). These are found mainly in Western Polesie, also known as Polesie Lubelskie (Wilgat et al. 1991; Michalczyk 1998; Chmielewski 2001). These lakes are considered karstic (carbonate karst). Another interesting place is the Połaniec Basin on the Małopolska Upland where karst landforms were formed on gypsum substrates – gypsum karst (Flis 1954; Chwalik-Borowiec 2013; Zieliński 2013).

Lakes which develop as a result of gypsum karst are interesting, though still little researched both in Poland and in the world.

In this study, karst lakes from Połaniec Basin have been analysed statistically, and then methods of statistical classification have been applied in order to attempt a typology of these lakes, to make a comparative analysis to establish whether, and if so, to what extent they are connected with each other.In other words, whether we can treat them as a uniform group.

The study involves selected lakes in the following three areas:(1) Staszów region, (2) the area east of Chmielnik, near Wola Zofiowska, and (3) between Chmielnik and Pińczów, near Szarbków(Fig. 1).
Permanent lakes of the biggest surface area in their group of lakes, and additionally those which have bathymetric planes, have been selected for these analyses.

## 2. Areas and lakes studied

The study area is situated in mesoregion of the Połaniec Basin at eastern part of the Nida Basin macroregion – both lie on the Małopolska Upland subprovince (Kondracki 2013).
So called covered, reproduced karst is a characteristic feature of the Basin's northern part. This is due tokarstifying layers, gypsum, which are usually covered by younger formations: Miocene clays of the Grabowiec and Krakowiec beds, also deposits of marginal facies (detrital) as well Pleistocene sands and tills (Senkowicz 1958; Walczowski 1968; Romanek 1982).In gypsum environment, karst lakes are quite common, but dry karst landforms, especially sinkholes, are even more numerous.
The biggest concentration of karst lakes is situated in the Staszów region. They are also present in the vicinity of Jarząbki and Szarbków. Some of these lakes are so close to each other that they form visible groups.

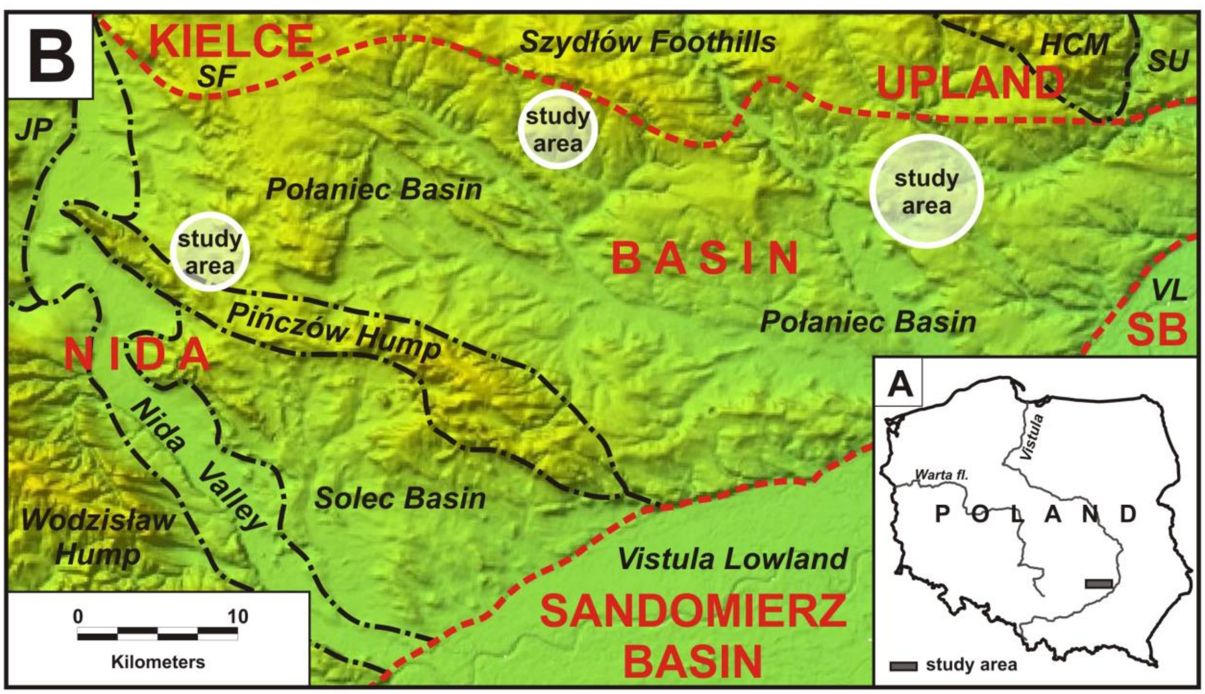


Figure 1. Location of study areas on the contour map of Poland (A).
Location of more numerous groups of lakes (B).

Red dashed lines – boundaries of macroregions (Kielce Upland, Nida Basin, Sandomierz Basin).
Black dash-dotted lines – boundaries of mesoregions (JP - Jędrzejów Plateau; SF - Szydłów Foothills, HCM – Holy Cross Mountains, SU – Sandomierz Upland, Połaniec Basin, Pińczów Hump, Wodzisław Hump, Nida Valley, Solec Basin, VL- Vistula Lowland).

The statistical analysis involved characteristic data of 23 lakes (Table 1; Fig. 2). Most of them are situated in the Staszów region, one lake is near Szarbków, and one lake near Wola Zofiowska. The original statistical material was prepared by Zieliński (2013) and it involved 25 lakes. In this study, part of the data (variables), those established and measured for 23 lakes, has been used, which has provided a uniform database. Morphometric parameters of lakes basins were calculated using the formulas included in the works of Skowron (2004) and Choiński (2007).

With regard to their form and basin shape, these lakes can be divided into(Zieliński 2013):

- lakes in single karst sinkholes, for example: Donica Lake, Łajba Lake, Drugi Staw, “Bez Nazwy” Lake, Pleban Lake;
- lakes in uvalas, for example: Duży Staw, Przedpole Lake, Rozlany Staw, Kacze Lake, Jasny Staw;
- lakes in bottoms of karst valleys which cut the surface of the Paleogene peneplanation surface, for example: Dziki Staw.

These lakes can also be grouped in the following way:

- those whose waters have no contact with karst formations, e.g. lakes in the Golejowskie Forests near Staszów, lakes near Jarząbki and lakes situated round Pleban Lake;
- those whose waters have contact with karstifying rock, e.g. some unnamed lakes in the Szaniec Plateau (Zieliński 2013).

The area of the lakes studied here is relatively small, ranging from 0.08 to 3.63 hectares(Zieliński 2013). Lakes situated in isolated karst sinkholes are especially small in size. Their small size notwithstanding, they are among the deepest natural lakes in the Świętokrzyski region.

It is worth noting that lakes in karst formations not always fill these formations to their full capacity. The present focus is on analysing lakes, without referring to karst formations themselves.

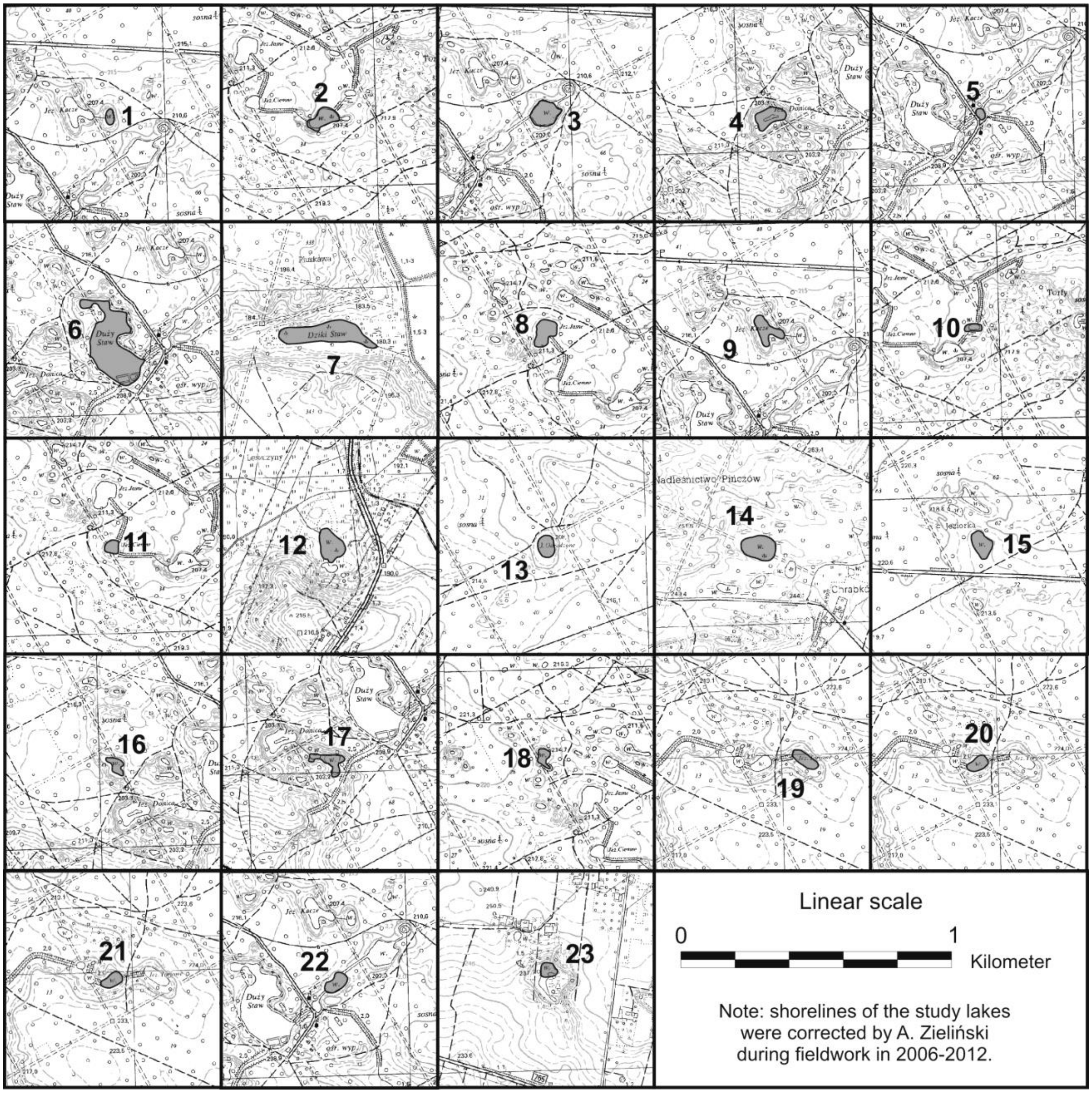


Figure 2. Location of the lakes studied in sections of topographical maps
(lakes numbering according to Table 1)

## 3. Methods

### 3.1. Parameters used in the analysis

For the statistical analyses, the following data have been collected from previous study of Zieliński (2013): data concerning the lakes' morphometry (surface area $P$, length $D$, maximum width $W$, mean width $W_m$, length ratio $\lambda$, length of shoreline $L$, development of shoreline length $K_1$) as well as parameters of lake basin (volume $V$, maximum depth $H_{\max}$, mean depth $H_m$, relative depth $H_r$, ratio of basin shape $W_g$ i.e. depth index, mean bottom slope $\alpha$ i.e. mean bottom inclination, index of lake basin permanence BPI i.e. basin permanence index). Altogether, information about 23 lakes was used according to 15 variables (Table 1). The Table shows the lakes in an alphabetical order. In order to ease search, the ordinal numbers in the first column of Table 1 refer to the lake numbers in Figure 2.

**Table 1**. Morphometric parameters of lakes and lake basins

| No | name | area ($P$) [m$^2$] | volume ($V$) [m$^3$] | length ($D$) [m] | maximum width ($W$) [m] | mean width ($W_m = P/D$) [m] | relative depth ratio ($C_R = H_{\max}/W_m$) [–] | lengthratio ($\lambda = D/W_m$) [–] |
|---|---|---|---|---|---|---|---|---|
| 1. | "Bez Nazwy" | 2100 | 2600 | 54 | 49 | 38.9 | 0.0913 | 1.39 |
| 2. | Ciemne | 4900 | 18500 | 116 | 72 | 42.2 | 0.0492 | 2.75 |
| 3. | CzwartyStaw | 9500 | 44100 | 143 | 103 | 66.4 | 0.1552 | 2.15 |
| 4. | Donica | 5900 | 14100 | 120 | 75 | 49.2 | 0.1598 | 2.44 |
| 5. | DrugiStaw | 1840 | 4000 | 68 | 44 | 27.1 | 0.1219 | 2.51 |
| 6. | DużyStaw | 36300 | 134000 | 425 | 156 | 85.4 | 0.1241 | 4.98 |
| 7. | DzikiStaw | 26000 | 32000 | 362 | 105 | 71.8 | 0.1039 | 5.04 |
| 8. | JasnyStaw | 7600 | 18800 | 127 | 84 | 59.8 | 0.0549 | 2.12 |
| 9. | Kacze | 6100 | 2600 | 120 | 76 | 50.8 | 0.1145 | 2.36 |
| 10. | lake on E from Ciemne | 800 | 1200 | 31 | 27 | 25.8 | 0.0669 | 1.2 |
| 11. | lake on W from Ciemne | 1800 | 1800 | 61 | 42 | 29.5 | 0.0334 | 2.07 |
| 12. | Łajba | 8200 | 14000 | 123 | 95 | 66.7 | 0.0568 | 1.85 |
| 13. | Odrodzone | 5700 | 17000 | 96 | 78 | 59.4 | 0.1268 | 1.62 |
| 14. | Pleban | 8600 | 13900 | 147 | 81 | 58.5 | 0.1255 | 2.51 |
| 15. | Przeciwpożarowe | 6600 | 8400 | 125 | 72 | 52.8 | 0.1215 | 2.37 |
| 16. | Przedpole | 3000 | 4600 | 82 | 77 | 36.6 | 0.0769 | 2.24 |
| 17. | RozlanyStaw | 2700 | 2300 | 78 | 79 | 34.6 | 0.109 | 2.25 |
| 18. | Szyja | 2500 | 3600 | 109 | 44 | 22.9 | 0.0645 | 4.75 |
| 19. | Torfowe I | 5100 | 14000 | 122 | 53 | 41.8 | 0.161 | 2.92 |
| 20. | Torfowe II | 3900 | 9500 | 96 | 67 | 40.6 | 0.0712 | 2.36 |
| 21. | Torfowe III | 2100 | 4200 | 58 | 46 | 36.2 | 0.1511 | 1.6 |
| 22. | TrzeciStaw | 4400 | 14400 | 95 | 57 | 46.3 | 0.0581 | 2.05 |
| 23. | Zofiówka | 3600 | 4800 | 83 | 75 | 43.4 | 0.0576 | 1.91 |

| No | shoreline length ($L$) [m] | developmentof shorelinelength ($K_1 = L/(2\sqrt{\pi P})$) [m·ha$^{-1}$] | maximumdepth ($H_{\max}$) [m] | meandepth ($H_m = V/P$) [m] | relative depth ($H_r = H_{\max}/\sqrt{P}$) [–] | ratio ofbasin shape ($W_g = H_m/H_{\max}$) [–] | meanbottom slope ($\alpha = \mathrm{atan}(h\sum l/P)$) [°] | index of basin permanence ($\mathrm{BPI} = V/L$) [m$^3$·m$^{-1}$] |
|---|---|---|---|---|---|---|---|---|
| 1. | 169 | 1.04 | 2.6 | 1.24 | 0.0567 | 0.4762 | 9 | 15.38 |
| 2. | 314 | 1.27 | 6.8 | 3.78 | 0.0971 | 0.5552 | 20 | 58.92 |
| 3. | 368 | 1.07 | 8.1 | 4.64 | 0.0831 | 0.5731 | 14 | 119.84 |
| 4. | 361 | 1.33 | 6.1 | 2.39 | 0.0794 | 0.3918 | 15 | 39.06 |
| 5. | 189 | 1.24 | 4.2 | 2.17 | 0.0979 | 0.5176 | 18 | 21.16 |
| 6. | 1095 | 1.62 | 7.8 | 3.69 | 0.0409 | 0.4733 | 9 | 122.37 |
| 7. | 860 | 1.5 | 2.4 | 1.23 | 0.0149 | 0.5128 | 4 | 37.21 |
| 8. | 387 | 1.25 | 4.6 | 2.47 | 0.0528 | 0.5378 | 12 | 48.58 |
| 9. | 413 | 1.49 | 2.5 | 0.43 | 0.032 | 0.1705 | 7 | 6.3 |
| 10. | 102 | 1.02 | 3.9 | 1.5 | 0.1379 | 0.3846 | 18 | 11.76 |
| 11. | 167 | 1.11 | 2.1 | 1 | 0.0495 | 0.4762 | 10 | 10.78 |
| 12. | 350 | 1.09 | 4.3 | 1.71 | 0.0475 | 0.3971 | 8 | 40 |
| 13. | 278 | 1.04 | 6.8 | 2.98 | 0.0901 | 0.4386 | 12 | 61.15 |
| 14. | 385 | 1.17 | 3.4 | 1.62 | 0.0367 | 0.4754 | 7 | 36.1 |
| 15. | 343 | 1.19 | 3 | 1.27 | 0.0369 | 0.4242 | 4 | 24.49 |
| 16. | 278 | 1.43 | 3.8 | 1.53 | 0.0694 | 0.4035 | 11 | 16.55 |
| 17. | 327 | 1.78 | 1.9 | 0.85 | 0.0366 | 0.4483 | 11 | 7.03 |
| 18. | 269 | 1.52 | 2.5 | 1.44 | 0.05 | 0.576 | 14 | 13.38 |
| 19. | 289 | 1.14 | 5.3 | 2.75 | 0.0742 | 0.5179 | 12 | 48.44 |
| 20. | 256 | 1.16 | 5.1 | 2.44 | 0.0817 | 0.4776 | 14 | 37.11 |
| 21. | 190 | 1.17 | 4.4 | 2 | 0.096 | 0.4545 | 16 | 22.11 |
| 22. | 236 | 1 | 7.4 | 3.27 | 0.1116 | 0.4423 | 17 | 61.02 |
| 23. | 238 | 1.12 | 2.5 | 1.33 | 0.0417 | 0.5333 | 16 | 20.17 |

Source: own materials, based on Zieliński (2013)
$\sum l$ – sum of length of all the isobaths; $h$ – contour line cut

### 3.2. Correlation analysis

As a preliminary investigation of relationship between of variables, simple correlation analyses of pairs of variables was carried out. Since the type of correlation is unknown, three different coefficients are calculated in each case – Pearson'$s r$, Spearman's $\rho$, and Kendall's $\tau$. The strength of correlation is assessed following the interpretation proposed by Evans (1996).

### 3.3. Cluster analysis

Inasmuch as some variables are direct transformations of other variables (e.g. length ratio of a lake is calculated by dividing its length by its maximum width), some of the data reflect close parameters (e.g. 'maximum width' and 'mean width' pair of variables), equivalent treatment of all the 15 variables is debatable.

For this reason, the cluster analysis (Gatnar and Walesiak 2004; Walesiak and Gatnar 2009) was carried out three separate times, on the basis of three base pairs: variables used in cluster analysis as well as the way the distance between objects is determined. The base pairs are as follows:

(P1) all the variables taken into account; distance determined on the basis of standardized values of all the variables;

(P2) part of variables taken into account; distance determined on the basis of values of these variables;

(P3) all the variables taken into account; distance determined on the basis of standardized, and then multiplied by weights of values of all the variables.

In each case, the analysis was based on a correspondingly multi-dimensional Euclidean metric, as distances between objects.

In the case of (P1) base pair, agglomerative clustering, followed by divisive clustering was performed – `hclust`, `agnes`, `diana` functions of `R Package` (R Core Team 2015) – with so called average distance taken as a distance measure between successively created (connected/divided) clusters. Parallelly, Caliński-Harabasz, silhouette, Dunn, an GAP indices as well as the Pearson gamma for partitions into $k$ clusters, with $k \in \{2,3,4,\ldots,22\}$, were determined in order to identify their optimum number (Korzeniewski 2014) with use of `cluster.stats` function (Hennig 2015).

In the case of (P2) base pair, all the same procedures were carried out. Variable typing was performed by heuristic identification of noisy variables, using `HINoV.Mod` function of `clusterSim` package – settings: partition into two clusters, Euclidean metric, distance between groups as mean distance (Walesiak and Dudek 2016). This method identified the following variables: 'mean bottom slope', 'mean depth', 'development of shoreline', 'length ratio', 'ratio of basin shape' as confounders, as a consequence, these variables were omitted. (The confounding variables are listed above in the order of the least to the most insignificant ones). The remaining variables, as was the case in (P1), served as the basis for agglomerative and divisive clustering. Likewise, indices were determined as was the case in (P1).

In turn, in the case of (P3) base pair, before carrying out an analogous procedure, an attempt at setting weights of all the variables was undertaken. To do this, roles of variables and objects were reversed and cluster analysis for variables was performed: agglomerative clustering (`hclust` function), with the distance between variables calculated as $2 - \tau$, where $\tau$ is the value of Kendall's tau coefficient for a given pair of variables, and with average distance between groups created. After that, values of so-called height, i.e. distance of successively included single variables to the groups already created, were raised to the 0.25 power, in order to weaken their excessive differentiation, and accepted as weights of variables.

**4. Preliminary investigation of relationship between pairs of variables**

Simple correlation analyses of pairs of variables showedquite a noticeable – very strong for Pearson's coefficient in the sense of Evans interpretation (Evans 1996) – relationship between:

- lakes' length and their maximum width ($r = 0.829$, $\rho = 0.751$, $\tau = 0.567$: values of Pearson's linear correlation coefficient, Spearman's rank correlation coefficient, and Kendall's tau coefficient respectively);
- lakes' area and their volume ($r = 0.892$, $\rho = 0.803$, $\tau = 0.652$);
- lakes' area and their length ($r = 0.987$, $\rho = 0.954$, $\tau = 0.851$);
- lakes' area and their shoreline length ($r = 0.980$, $\rho = 0.915$, $\tau = 0.758$);
- lakes' length and their shoreline length ($r = 0.986$, $\rho = 0.904$, $\tau = 0.755$);
- relative depth ratio of lakes and their mean bottom slope ($r = 0.841$, $\rho = 0.820$, $\tau = 0.693$).

Such relationships are most noticeable in the group of lakes situated in single karst sinkholes.

The shape of the karst basin correlates to the volume of water in a lake. However, lakes of a similar volume may differ considerably in terms of their surface area, which is the case in e.g. Trzeci Staw, Łajba and Pleban Lakes.

Interestingly, the increase in the maximum depth of the lakes analysed does not always result noticeablyin the increase in their volume ($r = 0.558$ – Evans' moderate, $\rho = 0.692$, $\tau = 0.564$). This is also the case of the dependence between the lake surface area and its maximum depth($r = 0.295$ – weak, $\rho = 0.349$, $\tau = 0.232$).However, the dependence between lake maximum depth and its mean depth is good ($r = 0.951$ – very strong,$\rho = 0.962$, $\tau = 0.849$): an increase in maximum depth, usually leads to an increase in mean depth. Relationship between the lakes' mean depth and their mean bottom slope is poor ($r = 0.4444$ – moderate, $\rho = 0.486$, $\tau = 0.312$).

In turn, the size of lake surface has negative relation with its mean bottom slope($r = – 0.448$ – Evans' moderate, $\rho = – 0.509$, $\tau = – 0.349$). Lakes of area of up to 0.5 ha have the biggest mean bottom slope.

**5. Results of cluster analysis**

In the case of (P1) base pair, the number of clusters was shown as 2 (20%indications), 3 (20% indications), 20 (20% indications), and 22 (40% indications). No other numbers of clusters were shown.

This suggests that the division into two or three clusters is the most justifiable among reasonable divisions.

Interestingly, all the clustering methods used determined the same two clusters: one consisting of Duży Staw and Dziki Staw, the other comprising all the remaining lakes.

These divisions are presented in the following dendrograms (Fig. 3-5):

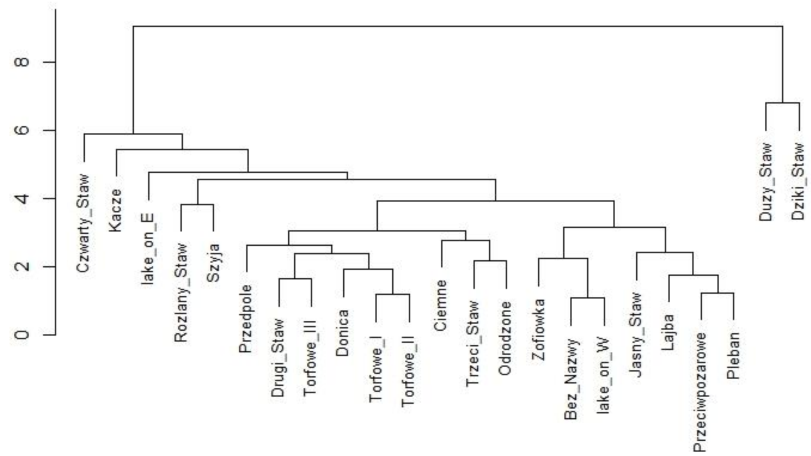


Figure 3. Dendrogram of lakes for (P1) base pair using 'Hclust' function

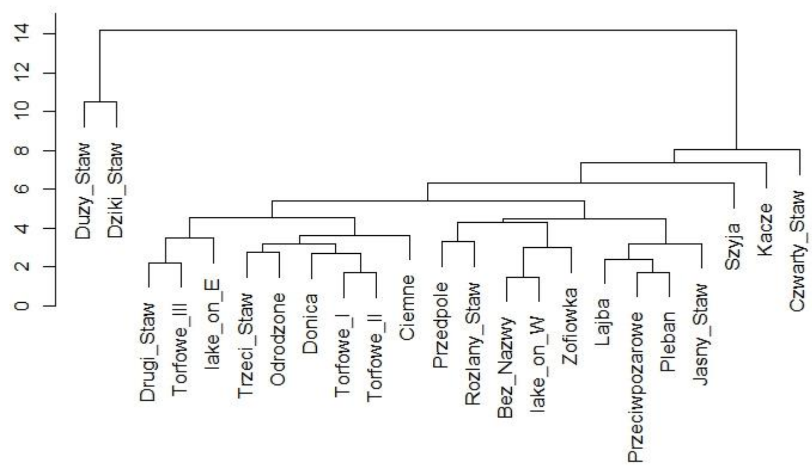


Figure 4. Dendrogram of lakes for (P1) base pair using 'Agnes' function

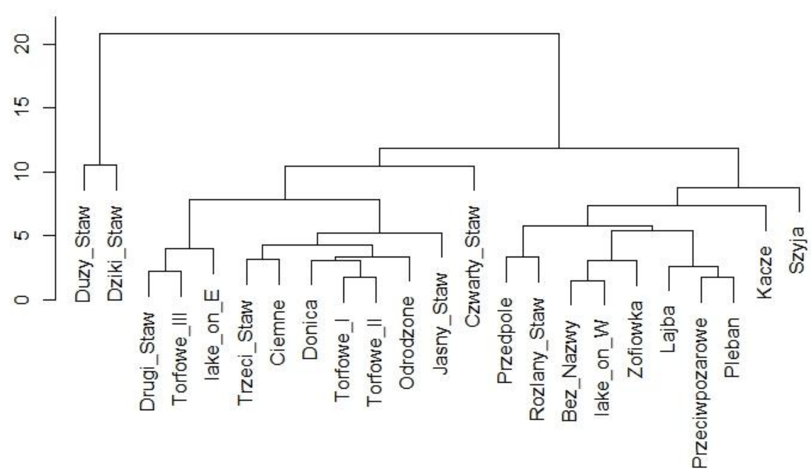


Figure 5. Dendrogram of lakes for (P1) base pair using 'Diana' function

In the division into three clusters, agglomerative methods resulted in identifying Duży Staw and Dziki Staw as two separate clusters, and all the remaining lakes as the third cluster. In turn, divisive methods put Duży Staw and Dziki Staw into one group, and the remaining lakes into two groups, roughly of a similar number of lakes, with Czwarty Staw and Szyja Lake, respectively, standing out the most.

It is worth noting that the option of dividing the lakes into 20 or 22 clusters may suggest that the set of the lakes analysed here, albeit with the exception of Duży Staw and Dziki Staw, and, may be, except Czwarty Staw and Szyja Lake as well, despite being quite homogeneous in terms of the variables that describe the lakes, is still not homogeneous enough for the lakes to be considered strongly similar to each other.

In the case of (P2) base pair, the number of clusters was shown as 2 (20% indications), 3 (20% indications), 19 (40% indications), and 22 (20% indications).No other numbers of clusters were shown.

In the case of two-cluster division, all the clustering methods used determined the same two clusters: one consisting of Duży Staw, and the other comprising all the remaining lakes with Dziki Staw, and Czwarty Staw, consecutively, ‘standing out’ the most.

The three-cluster division corresponds to that of (P1): consistent division into: 1. Duży Staw, 2. Dziki Staw, 3. all the remaining lakes. Interestingly, in the last group, Czwarty Staw, mentioned in (P1),is the one that also ‘stands out’ most.

It is necessary to point out that the omission of a number of variables – which was assumed in the case of (P2) base pair – was solely due to the specificity of the heuristic identification of noisy variables method, and, in the Authors’ opinion, this led to the omission of quite important variables that describe lakes, especially in the case of the ratio of basin shape variable.

Finally, the case of (P3) base pair starts with reversed clustering problem. Thus, the results of the variable grouping is shown in the dendrogram in Fig. 6.It is worth noting that the biggest weight is given to the ‘ratio of basin shape’ variable, which has been eliminated by the heuristic method as the most insignificant. The current weight given to this variable seems to fully reflect the importance ascribed to it intuitively.

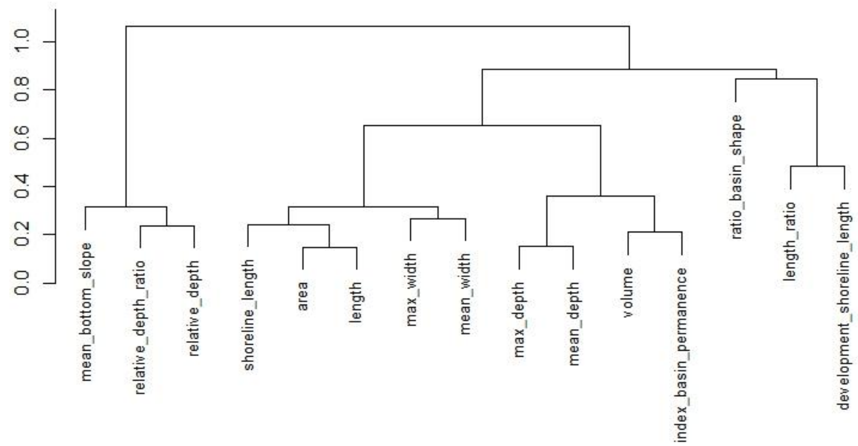


Figure 6. Dendrogram of variables for (P3) base pair

In this case as well, the numbers of clusters were similar, and they were as follows: 2 (30% indications), 4 (10% indications), 20 (40% indications), and 22 (20% indications).

The division into two clusters, the most justifiable one among reasonable divisions, determined the same two clusters as was the case in (P1) and (P2).In turn, the division into four clusters via divisive methods identify (1) Duży Staw and Dziki Staw, (2) Kacze Lake and Rozlany Staw, (3) Czwarty Staw with a group of 9 other lakes, (4) Szyja Lake with a group of 8 remaining lakes.

Further, analogous procedures were performed for (P1), (P2), and (P3) again, with the Euclidean distance replaced by the Manhattan distance. In these cases, very similar results were obtained, both for the suggested numbers of clusters and for the object grouping itself. Dominant reasonable divisions into two clusters are in each case identical to those with the Euclidean distance. The divisions into three clusters are also of the same types as before, albeit division into (1) Duży Staw, (2) Dziki Staw, (3) the rest of the lakes (with Czwarty Staw ‘standing out’ most) is absolutely predominant. There were no indications for divisions into four clusters.

Similarly, divisions into 19, 21 and 22 clusters were suggested. Thus, this time as well, we can see some kind of polarization of the numbers of clusters suggested.

The cluster analysis described above, and intentionally performed in many various ways, allows us to conclude that two lakes: Dziki Staw and Duży Staw clearly stand out from the other lakes. The remaining lakes are quite, albeit not an ideally homogeneous group. This homogeneity is disturbed by the suggested divisions into a big number of clusters, as well as by some lakes standing out of this group occasionally and forming separate clusters.

Such a perception of the objects studied is also confirmed by the way the points representing lakes are distributed in the system of coordinates for three primary principal components (*Principal Component Analysis*, PCA) determined on basis of the set of variables analysed. Obviously, the point distribution is dependent on choosing (P1), (P2) and (P3) as a starting point. Figure 7 shows (P1) the point distribution with the Euclidean distance, with PC1 and PC2 axes representing the first and second principal components respectively. The third component is shown by the size of the points: the smaller the point, the deeper it is from the PC1-PC2 plane. The points are numbered according to the lake numbers in Tab. 1.

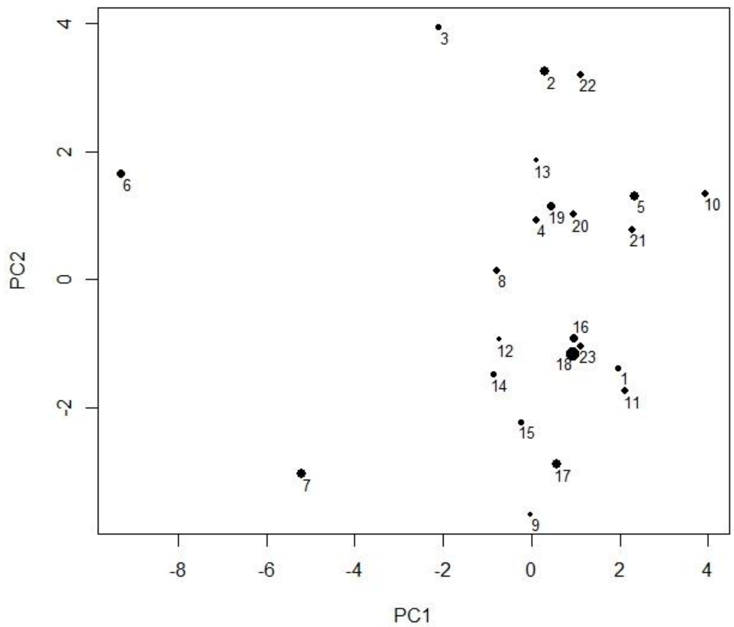


Figure 7. Lake distribution in the system of three principal components

## 6. Discussion

The analyzed lakes from the Połaniecka Basin area are generally small objects, and their characteristic feature is a relatively large depth and circular shape (Fig. 8). These parameters differ from other water bodies located in both Świętokrzyskie and other regions. Undoubtedly, geological determinants influence the unique specifics of the objects.

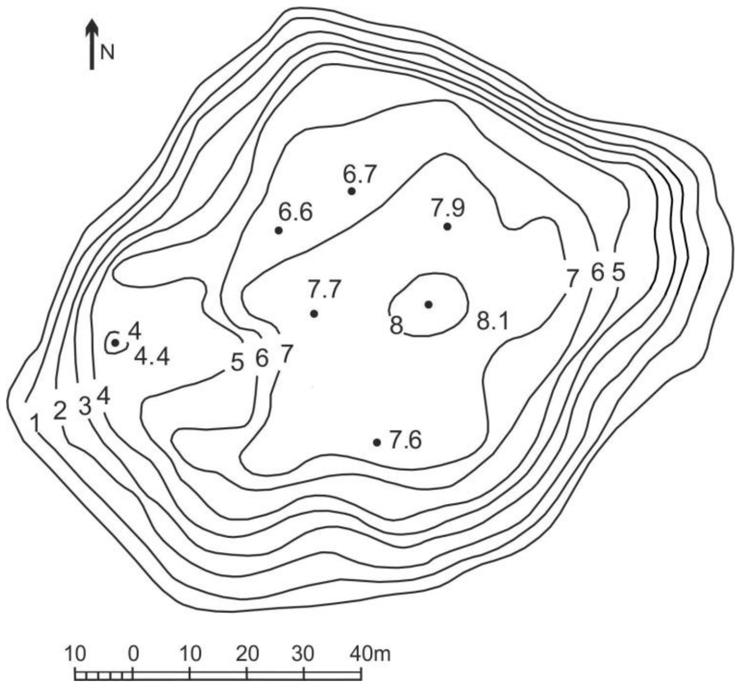


Figure 8. Bathymetric plan of Czwarty Staw, based on Zieliński (2013); numbers show depth in meters

Even smaller natural lakes are known from the area of the Kolbuszowa Plateau, from the Grodzisk region near Leżajsk (Wojtanowicz and Jóźwiakowska 1997). They occur primarily on the highest levels of the plateau, but also on its slopes, within the denudational valleys and in the valley of the Leszczynka stream. There are 82 lakes near Grodzisk, but only in two cases their longer axis exceeds 100 m. Most have from 10 to 80 m in length and from 10 to 40 m in width (Fig. 9). They are most often oval, less frequent circular. The lakes are relatively shallow: 20 of them have up to 1.0 m in depth, 24 up to 1.0 to 1.5 m, and 26 from 1.5 to 2.0 m. One lake is about 3.5 m deep, because that it was dredged by the owner of the ground on which it is located. Grodzisk lakes, which developed on the cover of silty deposits, i.e. located on the top and slopes of the Kolbuszowa Plateau, were formed in the late of LGM, together with degradation of long-term permafrost in the ground resulting from thermal karst (Wojtanowicz 1997). Research on these basins has shown that only 20 of them have water throughout the year (Jóźwiakowska 1997). Some of the lake basins (18) accumulate water only at the beginning of spring. In the rest of the lakes there have been anthropogenic changes, as a result of which 39 are already dried and 5 have been buried.

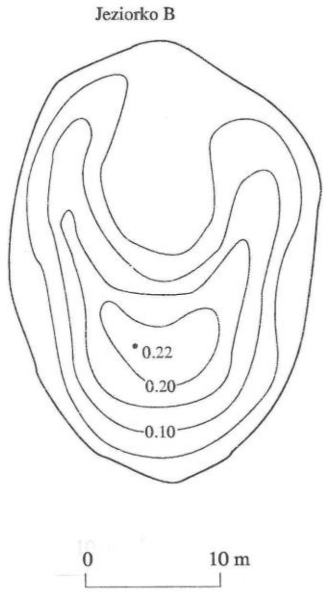


Figure 9. Bathymetric plan of Jeziorko B, based on Jóźwiakowska (1997); numbers show depth in meter

What in our climate conditions can be considered as unique, in other geographic locations, such as northern Norway, is something common (cf. Svensson 1969). In the circumpolar climates zone, in the subpolar climate type, occurrence of long-term permafrost provides the formation of characteristic oval lakes. The dimensions of these aquatic areas are often small, of the order of several dozen meters. They are shallow, usually up to 1 m deep. The occurrence of earth embankments around their bowls is characteristic.

In recent years, dune lakes have been described in the Świętokrzyskie region, and their assemblages called (aeolian) lakeland (Jaśkowski and Sołtysik 2003). This name was used rather not in a regional sense, but in the sense of landscape. The surface of these basins are sometimes larger than the described karst lakes, but their depth is small, up to 2 m.

Much larger aquatic areas occur in northern Poland, in lakelands formed within the glacial uplands (Choiński and Ptak 2009). Lakes there have (eg within the Greater Poland-Kujawy Lakeland, Mazurian Lakeland, Pomeranian Lakeland) hundreds of hectares, maximum depths of tens of meters, medium depths of a few to several meters, volume from several hundred thousand, up to several tens of millions of m3. The shapes of bowls of these lakes are different. There are compact bowls (melt lakes), elongated (gutter lakes), with a mixed shape (diversified origin). Recent research indicates water loss in lakes and, consequently, changes in the coastline, e.g. formation of peninsulas and islands (Choiński et al., 2016).

Interesting are the coastal lakes Jamno and Bukowo (Choiński et al., 2014). They are of ca. 2.2-1.6 thousand ha, water volumes of ca. 38-28 million m3, but at the same time very shallow, of average 1.7 m. Over the last century, the their area decreased by 8-14%, water volume 6-18%,with little loss pf depth.

It seems interesting to compare lakes from other regions of Poland with a different origin, with the karst lakes of the Połaniecka Basin, in terms such as presented in this work. For this more data should be collected, similar to those provided by Zieliński (2013). For the same reason, the authors cannot compare the discussed reservoirs with other sites located in areas with a similar geological structure.

This article also provides data on the morphometry of karst mire lakes.

## 7. Conclusions

The lakes selected for the statistical analysis are situated within the same mezoregion, Połaniec Basin. They are in the area of a similar geological structure and climate conditions, hence their homogeneous origin.

Correlation analysis showed a clear relationship between the lake length and its width. Such dependences are the most visible in the group of lakes in single karst sinkholes. The analysis also shows that the lake basin shape is a decisive factor of the volume of water in a given lake. However, lakes of a similar volume may differ in terms of their area. What is more, an increase in maximum depth does not always result in an increase in volume. This is also the case in dependence between the lake area and its maximum depth.

The most frequent reasonable indication of the cluster analysis was for grouping lakes within two sets. One of them consists of Duży Staw and Dziki Staw, and the other set comprises the remaining 21 lakes. In this method, in the division into three clusters (the division occurring less frequently), Duży Staw, Dziki Staw, and Czwarty Staw stood out the most, creating either two groups or three separate groups. In turn, the divisions of the lakes analysed into 19–22 clusters occurred more often, but it does not seem to be reasonable taking into account such divisions, since they nearly reached the number of the lakes.

The cluster analysis performed in many various ways shows that Dziki Staw and Duży Staw stand out the most from the other lakes. The other lakes are close to each other, but they do not form an ideally uniform group.

## References


CHMIELEWSKI T. J., 2001, Pojezierze Łęczyńsko-Włodawskie. Przekształcenia struktury ekologicznej krajobrazu i uwarunkowania zagospodarowania przestrzennego. Lublin University of Technology Press, Lublin, 1-146.

CHOIŃSKI A., 1995, Katalog jezior Polski. Cz. 3. Jeziora Wielkopolsko-kujawskie i jeziora na południe od linii zasięgu zlodowacenia bałtyckiego. Adam Mickiewicz University Press, Poznań, 1-149.

CHOIŃSKI A., 2007, Limnologia fizyczna Polski. Adam Mickiewicz University Press, Poznań, 1-548.

CHOIŃSKI A., PTAK M., 2009, Lake infill as the main factor leading to lake's disappearance. Polish Journal of Environmental Studies, 18, 3: 347-352.

CHOIŃSKI A., PTAK M., ŁAWNICZAK A., 2016, Changes in water resources of Polish lakes as influenced by natural and anthropogenic factors. Polish Journal of Environmental Studies, 25, 5: 1883-1890.

CHOIŃSKI A., PTAK M., STRZELCZAK A., 2014, Present-day evolution of coastal lakes based on the example of Jamno and Bukowo (the Southern Baltic coast). Oceanological and Hydrobiological Studies, 43, 2: 178-184.

CHWALIK-BOROWIEC A., 2013, Rzeźba terenu parków krajobrazowych Ponidzia. Kielce, 1-156.

EVANS J. D., 1996, Straightforward statistics for the behavioral sciences. Brooks/Cole Publishing, Pacific Grove, CA.

FLIS J., 1954, Kras gipsowy Niecki Nidziańskiej. Prace Geograficzne, 1.

GATNAR E., WALESIAK M. (ed.), 2004, Metody statystycznej analizy wielowymiarowej w badaniach marketingowych. Wrocław University of Economics Press, Wrocław.

HENNIG C., 2015, fpc: Flexible Procedures for Clustering. R package version 2.1-10. https://CRAN.R-project.org/package=fpc.

JAŚKOWSKI B., SOŁTYSIK R., 2003, The origin and age of the Świętokrzyski Lake Discrit. Limnological Review, 3: 101-106.

JÓŹWIAKOWSKA E., 1997, Współczesny stan i rozwój jeziorek w okolicy Grodziska. [in:] Łanczont M. (red.), Seminarium terenowe, Glacjał i peryglacjał Kotliny Sandomierskiej i przedgórza Karpat w okolicy Przemyśla, Krasiczyn. UMCS Press, Lublin: 119-123.

KONDRACKI J., 2013, Geografia regionalna Polski. PWN Press, Warszawa, 1-468.

KORZENIEWSKI J., 2014, Indeks wyboru liczby skupień w zbiorze danych. Przegląd Statystyczny, 61, 2: 169-180.

MICHALCZYK Z., 1998, Stosunki wodne Pojezierza Łęczyńsko-Włodawskiego. [in:] M. Harasimiuk, Z. Michalczyk, M. Turczyński (ed.) Jeziora łęczyńsko-włodawskie. Monografia przyrodnicza. Biblioteka Monitoringu Środowiska. UMCS Press, Lublin, 55-72.

R Core Team, 2015, R: A language and environment for statistical computing. R Foundation for Statistical Computing, Vienna, Austria. URL http://www.R-project.org/.

ROMANEK A., 1982, Szczegółowa mapa geologiczna Polski w skali 1:50 000, ark. Chmielnik (885) wraz z objaśnieniami. Wydawnictwa Geologiczne, Warszawa.

SENKOWICZ E., 1958, Szczegółowa mapa geologiczna Polski w skali 1:50 000, ark. Pińczów (884). Wydawnictwa Geologiczne, Warszawa.

SKOWRON R., 2004, Description of lake basin in the light of selected morphometric indicators. [in:] Machowski R., Rzętała M. (ed.), Limnological Review, 4: 233 – 240.

SVENSSON H., 1969, A type of circular lakes in northernmost Norway. Geografiska Annaler. Series A, Physical Geography, 51, 1/2: 1-12.

WALCZOWSKI A., 1968, Objaśnienia do Szczegółowej mapy geologicznej Polski w skali 1:50 000, ark. Staszów (886). Wydawnictwa Geologiczne, Warszawa.

WALESIAK M., DUDEK A., 2016, cluster Sim: Searching for Optimal Clustering Procedure for a Data Set. R package version 0.45-1. https://CRAN.R-project.org/package=clusterSim.

WALESIAK M., GATNAR E. (ed.), 2009, Statystyczna analiza danych z wykorzystaniem programu R. PWN Press, Warszawa.

WILGAT T., MICHALCZYK Z., TURCZYŃSKI M., WOJCIECHOWSKI K. H., 1991, Jeziora Łęczyńsko-Włodawskie. Studia Ośrodka Dokumentacji Fizjograficznej, 1: 23-140.

WOJTANOWICZ J., 1997, Geneza i wiek jeziorek grodziskich. [in:] Łanczont M. (red.), Seminarium terenowe, Glacjał i peryglacjał Kotliny Sandomierskiej i przedgórza Karpat w okolicy Przemyśla, Krasiczyn. UMCS Press, Lublin: 118.

WOJTANOWICZ J., JÓŹWIAKOWSKA E., 1997, Jeziorka okolic Grodziska - występowanie, charakterystyka. [in:] Łanczont M. (red.), Seminarium terenowe, Glacjał i peryglacjał Kotliny Sandomierskiej i przedgórza Karpat w okolicy Przemyśla, Krasiczyn. UMCS Press, Lublin: 113-117.

ZIELIŃSKI A., 2013, Rozwój jezior krasowych w Niecce Połanieckiej. Jan Kochanowski University Press, Kielce, 1-186.